IES/ISS EXAM,2014

A-HDR/HRR-N-TUA

STATISTICS

Paper I

Time Allowed : Three Hours *Maximum Marks : 200*

INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions :

There are EIGHT questions divided under TWO sections.

Candidate has to attempt FIVE questions in ALL.

All the parts in the ONLY question in Section A are compulsory.

Questions no. 1 and 5 are compulsory and out of the remaining, THREE are to be attempted choosing at least ONE from each Section.

The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Candidates should attempt questions/parts as per the instructions given in the Section.

All parts and sub-parts of a question are to be attempted together in the answer book.

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the answer book must be clearly struck off.

Answers must be written in ENGLISH only.

A Normal Distribution table and a 't' table are attached with the question paper.

SECTION A

1. Answer ***all*** of the following : *5×8=40*

(a) A positive integer X is selected at random from the first 50 natural numbers. Calculate $P\left(X + \frac{96}{X} > 50\right)$. *8*

(b) Suppose that all the four outcomes O_1, O_2, O_3 and O_4 of an experiment are equally likely. Define $A = \{O_1, O_4\}$, $B = \{O_2, O_4\}$ and $C = \{O_3, O_4\}$. What can you say about the pairwise independence and mutually independence of the events A, B and C ? *8*

(c) The marginal distributions of X and Y are given in the following table :

y \ x	1	2	Total
3			$\frac{1}{4}$
4			$\frac{3}{4}$
Total	$\frac{1}{2}$	$\frac{1}{2}$	1

If the covariance between X and Y is zero, find the cell probabilities and see whether X and Y are independent. *8*

(d) Show that for 40,000 throws of a balanced coin, the probability is at least 0·99 that the proportion of heads will fall between 0·475 and 0·525. *8*

(e) If $f_X(x)$ be the probability density function of a $N(\mu, \sigma^2)$ distribution, then show that

$$\int_L^U x f_X(x)\, dx = \mu\, [\Phi(U') - \Phi(L')] - \sigma\, [\phi(U') - \phi(L')]$$

where $L' = \frac{L - \mu}{\sigma}$, $U' = \frac{U - \mu}{\sigma}$ and $\phi(z)$ and $\Phi(z)$ are the probability density function and distribution function of the standard normal distribution respectively. *8*

2. (a) $X_1, X_2, \ldots, X_N$ are independently, identically distributed random variables. Define $S_N = X_1 + X_2 + \ldots + X_N$, where N is a random variable independent of X_i, $i = 1, 2, \ldots N$.

Show that the moment generating function (mgf) of S_N is

$$M_{S_N}(t) = M_N(\log M_X(t))$$

where $M_Y(t)$ is the mgf of a random variable Y.

Hence find the mgf of S_N when N follows a Poisson distribution with parameter λ and X_i follows an exponential distribution with mean parameter θ, $i = 1(1)N$. *15*

(b) If $f_X(x)$ be the probability density function of a lognormal distribution, show that

$$\int_L^U x^k f_X(x)\,dx = e^{k\mu + \frac{1}{2}k^2\sigma^2} \left[\Phi(U_k) - \Phi(L_k)\right]$$

where $L_k = \frac{\log L - \mu}{\sigma} - k\sigma$ and

$U_k = \frac{\log U - \mu}{\sigma} - k\sigma$ and $\Phi(z)$ is the distribution function of the standard normal distribution. Hence find E(X) and V(X). *15*

(c) In a lottery 1000 tickets are sold and the cost of a ticket is ₹ 10. The lottery offers a first prize of ₹ 1,000, two second prizes of ₹ 500 each, and three third prizes of ₹ 100 each. A person purchases a ticket. If X denotes the amount he may get, find E(X) and V(X). *10*

3. (a) n balls are distributed among r cells at random, each cell being free to receive any number of balls. Calculate the probability that a particular cell contains k balls ($k \leq n$) when (i) balls are distinguishable, and (ii) balls are non-distinguishable. *15*

(b) Write down the probability mass function of a trinomial distribution. Obtain the moment generating function of the distribution. Show that the correlation coefficient of any two variables is negative. Why ? *15*

(c) Prove that two events cannot be incompatible and independent simultaneously. *10*

4. (a) $\{X_n\}$ is a sequence of independent random variables. Show that $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{L} X$ where X is a random variable. Is the converse true ? *15*

(b) There are three identical bags U_1, U_2 and U_3. U_1 contains 3 red and 4 black balls; U_2 contains 4 red and 5 black balls; U_3 contains 4 red and 4 black balls. One bag is chosen at random; a ball is drawn at random from the chosen bag and it is found to be red. Find the probability that the first bag is chosen. *15*

(c) Let P(s) be the probability generating function associated with a non-negative, integer valued random variable X. Show that

$$\sum_{n=0}^{\infty} s^n \, P(X \le n) = \frac{P(s)}{1-s}. \qquad 10$$

SECTION B

5. Answer ***all*** of the following : *5×8=40*

(a) The runs scored by two batsmen A and B in five cricket matches were as follows :

Batsman A :	50	60	100	70	20
Batsman B :	120	100	30	20	40

Discuss the consistency and efficiency of the batsmen. *8*

(b) Given the following correlation matrix of order 3 × 3

$$R = \begin{bmatrix} 1 & 0\cdot5708 & 0\cdot6735 \\ & 1 & 0\cdot4487 \\ & & 1 \end{bmatrix}$$

Calculate (i) $r_{12.3}$ (ii) $r_{1.23}$. *8*

(c) Consider two samples as follows :

Sample 1 : 1, 4, 7, 9, 16, 17, 22, 24

Sample 2 : 2, 6, 10, 12, 18, 20, 26, 28, 32

Test whether the examples have come from the same population by Wilcoxon-Mann-Whitney test. [Given value of Z for $\alpha = 0\cdot05 = 1\cdot645$, where Z is N(0, 1)] *8*

(d) Define power of a test and discuss its role in selecting the best test. Describe a test procedure for testing equality of means of two independent normal populations, when standard deviations are equal but unknown for small samples. *8*

(e) Given the following values of the function $y = f(x)$; evaluate $f(4)$ and also find x for which $f(x) = 25$.

$$f(1) = 10, \; f(2) = 15, \; f(3) = 42.$$ *8*

6. (a) Describe a test procedure for testing the null hypothesis $H_0 : \mu_x - \mu_y = \delta_0$ (a particular value) for a bivariate normal population $B_N(\mu_x, \mu_y \; \sigma_x^2, \sigma_y^2 \; \rho)$ when (i) σ_x^2, σ_y^2 and ρ are known; (ii) σ_x^2, σ_y^2 and ρ are unknown. *15*

(b) Let $X_1, X_2, \ldots, X_n$ be n independently identically distributed random variables following an exponential distribution with mean parameter θ. Obtain the distributions of (i) Maximum $(X_1, X_2, \ldots, X_n)$; (ii) Minimum $(X_1, X_2, \ldots, X_n)$ and (iii) Median for $n = 2m + 1$, $(m > 0)$. *15*

(c) A medicine supposed to have effect in preventing TB was treated on 500 individuals and their records were compared with the records of 500 untreated individuals as follows. Study the effectiveness of medicine by calculating (i) Yule's coefficient of association (ii) Yule's coefficient of colligation. *10*

	NO-TB	TB
Treated	252	248
Untreated	224	276

7. (a) Distinguish between inter-class correlation and intra-class correlation.

The height (in cm) of three brothers belonging to each of three families are recorded below. Compute the intra-class correlation coefficient. *15*

Family	Heights of brothers (in cm)		
I	158·0	161·5	161·3
II	166·6	170·7	169·4
III	163·6	163·3	164·1

(b) Discuss the role of asymptotic relative efficiency in judging the relative preference of two tests. Following are the yields (in kg) of a crop recorded from an experiment with median (M) = 20 :

15·4, 16·4, 17·3, 18·2, 19·2, 20·9, 22·7, 23·6, 24·5

Test the null hypothesis

$H_0 : M = 20$ against $H_1 : M \neq 20$ at $\alpha = 0{\cdot}05$.

[You are given $P[X \leq 4] = 0{\cdot}50$] *15*

(c) The force of mortality is defined as $\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx}$ when l_x is the number of persons at exact age x (in years) in any year of time. Given the following table, find a value of μ_{50}. *10*

Age (x)	:	50	51	52	53
l_x	:	73,499	72,724	71,753	70,599

8. (a) Explain the procedure for testing the hypothesis of equality of variances of two independent normal populations when population means are unknown. Write down the sampling distribution of the statistic. A sample of size 10 is drawn from each of two uncorrelated normal populations. Sample means and variances are :

1^{st} population : mean = 7, variance = 26

2^{nd} population : mean = 4, variance = 10

Test at 5% level of significance whether the first population has greater standard deviation than that of the second population.

[Given $F_{0{\cdot}05;\, 9,9} = 3{\cdot}18$] *15*

(b) Let X follow a binomial distribution B(n, P). Explain the test procedure for

$H_0 : P = P_0$ against $H_1 : P > P_0$

when the sample size is (i) small, and (ii) large.

It is desired to use sample proportion p as an estimator of the population proportion P, with probability 0·95 or higher, that p is within 0·05 of P. How large should sample size (n) be ? *15*

(c) Using Euler's method, compute the values of y correct upto 4 places of decimal for the differential equation $\frac{dy}{dx} = x + y$ with initial condition $x_0 = 0,\ y_0 = 1,$ taking $h = 0{\cdot}05$. *10*

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t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tail	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

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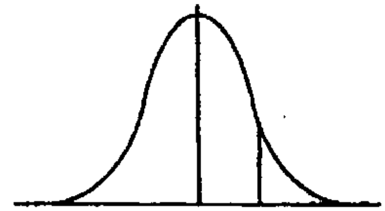
Normal Distribution Table



	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998